

Minimal no-radiation approximation to Einstein's field equations

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An approximation to Einstein's field equations in the Arnowitt-Deser-Misner canonical formalism is presented, which corresponds to the magnetohydrodynamics approximation in electrodynamics. It results in coupled elliptic equations which represent the maximum of elliptic-type structure of Einstein's theory and naturally generalizes previous conformal-flat truncations of the theory. The Hamiltonian, in this approximation, is identical with the nondissipative part of the Einsteinian one through the third post-Newtonian order. The proposed scheme, where stationary spacetimes are exactly reproduced, should be useful to construct *realistic* initial data for general relativistic simulations as well as to model astrophysical scenarios, where the gravitational radiation reaction can be neglected.

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I. INTRODUCTION

In electrodynamics, to model quasistationary scenarios accurately, it is customary to drop the Maxwell displacement current in the Maxwell-Ampère law of Maxwell's equations. This approximation is often referred to as the magnetohydrodynamics (MHD) approximation as it is mainly implemented in MHD computations dealing, mostly, with astrophysical plasmas. A strictly analogous approximation to Einstein's general relativity should prove very useful to model nonstationary situations involving relativistic self-gravitating systems.

In Einstein's theory of gravity, the conformal-flat approximations played a major role within numerical relativity as they resulted in simple elliptic equations [1]. However, in the past, it was pointed out that these approximations are rather crude for highly nonspherical objects such as rotating disks of dust or compact binary systems [2,3]. Therefore, in the following, we shall propose an elliptic-type approximation which goes far beyond conformal-flat schemes. Whereas conformal-flat slices do not exist under stationarity conditions [4,5], in the new approach Einstein stationarity is reproduced exactly. In a post-Newtonian setting, the new approach, in general, does not reproduce all even-order post-Newtonian terms (of course, the odd terms are not part of the new approach) beyond the second post-Newtonian order. However, at the third post-Newtonian order of approximation, it still gives the equations of motion, derivable from the full Einstein theory [6,7]. We observe that the very recently proposed gravitoelastic approximation is quite different from our minimal no-radiation approximation, especially for the choice of the dropped and the kept time derivatives in the evolution equations [8]. Our approximation also results in elliptic equations, which should be solvable employing LORENE, a highly efficient numerical library, based on spectral methods, developed by the Meudon group [9].

II. EINSTEIN'S THEORY IN THE ARNOWITT-DESER-MISNER FORMALISM

In the Arnowitt-Deser-Misner (ADM) formulation of general relativity, the spacetime line element in the (3+1) decomposed form is given by

$$ds^2 = -\alpha^2 c^2 dt^2 + \gamma_{ij}(dx^i + \beta^i c dt)(dx^j + \beta^j c dt) \quad (i, j = 1, 2, 3), \quad (1)$$

where α is the lapse function, β^i the shift vector, γ_{ij} the induced metric on a three-dimensional spatial slice $\Sigma(t)$, parametrized by the time coordinate t , and c the velocity of light. The three-metric and its canonical conjugate $(c^3/16\pi G)\pi^{ij}$, which is a contravariant symmetric tensor density of weight +1, satisfy the Hamiltonian and momentum constraints [10]

$$\gamma^{1/2} R = \frac{1}{2\gamma^{1/2}}(2\pi_j^i \pi_i^j - \pi^2) + \frac{16\pi G}{c^4} \gamma^{1/2} \alpha^2 T^{00}, \quad (2)$$

$$-2\pi_{i,j}^j + \pi^{kl} \gamma_{kl,i} = \frac{16\pi G}{c^4} \gamma^{1/2} \alpha T_i^0, \quad (3)$$

where R is the curvature scalar of $\Sigma(t)$, γ the determinant of γ_{ij} , $\pi_j^i = \gamma_{jk} \pi^{ik}$, and $\pi = \pi_i^i$. T^{00} and T_i^0 are the components of the unspecified four-dimensional stress-energy tensor for the matter, $T^{\mu\nu}$. The canonical conjugate π^{ij} is related to K_{ij} , the extrinsic curvature of $\Sigma(t)$, by $\pi^{ij} = -\gamma^{1/2}(\gamma^{il} \gamma^{jm} - \gamma^{ij} \gamma^{lm}) K_{lm}$, where γ^{il} is the inverse metric of γ_{ij} . In the above equations, a partial derivative is denoted by a comma, and G is the Newtonian gravitational constant. We note that the more familiar form for the left hand side of Eq. (3), namely, $\pi_{i|j}^j$, where $|$ stands for the three-dimensional covariant derivative, is expanded as $\pi_{i,j}^j - \frac{1}{2} \pi^{kl} \gamma_{kl,i}$, using the fact that π_i^j is a mixed tensor density of weight +1. The three-metric and its canonical conjugate evolve in accordance with the following evolution equations [10]:

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$$\begin{aligned}
\pi^{ij}_{,0} = & -\frac{1}{2} \alpha \gamma^{1/2} (2R^{ij} - \gamma^{ij}R) + \frac{1}{4} \alpha \gamma^{-1/2} \gamma^{ij} (2\pi^m_n \pi^m_n - \pi^2) \\
& - \alpha \gamma^{-1/2} (2\pi^{im} \pi^j_m - \pi \pi^{ij}) + \gamma^{1/2} (\alpha^{ij} - \gamma^{ij} \alpha^m_m) \\
& + (\pi^{ij} \beta^m_m)_{|m} - \pi^{mj} \beta^i_{|m} - \pi^{mi} \beta^j_{|m} \\
& + \frac{8\pi G}{c^4} \alpha \gamma^{1/2} \gamma^{il} \gamma^{jm} T_{lm}
\end{aligned} \quad (4)$$

and

$$\gamma_{ij,0} = \alpha \gamma^{-1/2} (2\pi_{ij} - \gamma_{ij}\pi) + \beta_{i|j} + \beta_{j|i}, \quad (5)$$

where R_{ij} is the Ricci tensor associated with $\Sigma(t)$. In this paper, we raise and lower indices on three-dimensional objects with γ^{ij} and γ_{ij} , respectively.

The ADM coordinate conditions which generalize the isotropic Schwarzschild metric read

$$\pi^{ii} = 0, \quad (6)$$

where, and from here onward, repeated covariant or contravariant indices imply the usage of the Einstein summation convention and

$$\gamma_{ij} = \psi^4 \delta_{ij} + h^{TT}_{ij}, \quad (7)$$

where ψ is a conformal scalar and h^{TT}_{ij} the transverse-traceless (TT) part of the three-metric γ_{ij} with respect to the Euclidean three-metric δ_{ij} . By definition, h^{TT}_{ij} satisfies $h^{TT}_{ii} = h^{TT}_{ij,j} = 0$. The restriction $h^{TT}_{ij} = 0$ gives a simple expression for the three-dimensional curvature scalar, $\gamma^{1/2}R = -8\psi\Delta\psi$, where Δ stands for the three-dimensional Euclidean Laplacian. The differential equation, used for gauge fixing γ_{ij} , reads

$$3\gamma_{ij,j} - \gamma_{jj,i} = 0. \quad (8)$$

Taking into account the gauge condition for π^{ij} , namely, Eq. (6), the following decomposition can be achieved:

$$\pi^{ij} = \tilde{\pi}^{ij} + \pi^{ij}_{TT}, \quad (9)$$

where $\tilde{\pi}^{ij}$ denotes the longitudinal part of π^{ij} . It may be expressed as

$$\tilde{\pi}^{ij} = \pi^j_{,i} + \pi^i_{,j} - \frac{2}{3} \delta_{ij} \pi^m_m, \quad (10)$$

which implies $\pi^{ij}_{,j} = \tilde{\pi}^{ij}_{,j} = \Delta \pi^i + \frac{1}{3} \pi^i_{,j,j}$, suggesting that Eq. (3) can be used to compute an elliptic equation for π^i . The TT part π^{ij}_{TT} , namely, $(c^3/16\pi G)\pi^{ij}_{TT}$, is the canonical conjugate to h^{TT}_{ij} , which gives the independent field degrees of freedom. The Hamiltonian, which generates the time evolution of the independent degrees of freedom of the system (matter plus gravitational field), is given by [10]

$$H[V_M, h^{TT}_{ij}, \pi^{ij}_{TT}] = -\frac{c^4}{2\pi G} \int d^3x \Delta \psi [V_M, h^{TT}_{ij}, \pi^{ij}_{TT}], \quad (11)$$

where V_M denotes the (nonspecified) matter variables. If the matter system consists of black holes, the matter variables V_M in our paper enter via boundary conditions at the apparent horizons and at spacelike infinity.

Equations (6) and (7) result in the covariant trace of π^{ij} of the form $\pi = \pi^{ij} h^{TT}_{ij}$. Taking into account the space-asymptotic properties $\pi^{ij} \sim 1/r^2$ and $h^{TT}_{ij} \sim 1/r$, the gauge condition Eq. (6) turns out to mean asymptotic maximal slicing. The gauge conditions Eqs. (6) and (7), or (8), are very close to the well-known Dirac gauge conditions. For example, the condition, given by Eq. (8), is identical with the corresponding Dirac gauge condition to linear order in $\gamma_{ij} - \delta_{ij}$.

The functions ψ and π^i , and hence $\tilde{\pi}^{ij}$, are determined using the Hamiltonian and momentum constraints, given by Eqs. (2) and (3), by elliptic equations. The elliptic equations for the (scalar) lapse α and the (vector) shift functions β^i result from the Eqs. (4) and (5), applying the coordinate conditions Eqs. (6) and (7), respectively. The explicit Poisson-type equations for α and β^i can be derived from

$$\begin{aligned}
\gamma^{ii} \alpha^m_{|m} - \alpha^{ii} = & -\alpha(R^{ii} - \gamma^{ii}R) - 2\alpha \gamma^{-1} \pi^{im} \pi^i_m \\
& - 2\gamma^{-1/2} \pi^{im} \beta^i_{|m} \\
& + \frac{8\pi G}{c^4} \alpha (\gamma^{il} \gamma^{jm} T_{lm} - \gamma^{ii} \alpha^2 T^{00}),
\end{aligned} \quad (12)$$

where use has been made of Eq. (2). The elliptic equation for β^i results from

$$\begin{aligned}
(\beta_{i|j})_{,j} + (\beta_{j|i})_{,i} - \frac{2}{3} (\beta_{j|j})_{,i} = & -[\alpha \gamma^{-1/2} (2\pi_{ij} - \gamma_{ij}\pi)]_{,j} \\
& + \frac{1}{3} [\alpha \gamma^{-1/2} (2\pi_{jj} - \gamma_{jj}\pi)]_{,i}.
\end{aligned} \quad (13)$$

The functions h^{TT}_{ij} and π^{ij}_{TT} in general follow from the evolution equations, given by Eqs. (4) and (5). Alternatively, h^{TT}_{ij} and π^{ij}_{TT} may be obtained from the Hamiltonian Eq. (11), via the corresponding Hamilton equations.

III. THE MINIMAL NO-RADIATION APPROXIMATION

Motivated by the MHD approximation in electrodynamics, a minimal truncation of the Einstein theory, in view of suppressing radiation emission, is achieved by putting *only* the TT part for the evolution equation for π^{ij} , given by Eq. (4), to zero. The TT part of Eq. (4) may be denoted as

$$\pi^{ij}_{TT,0} = A^{ij}_{TT}, \quad (14)$$

where $A_{TT}^{ij} \equiv A^{ij} - A_L^{ij}$. The expression for A^{ij} , obtainable from the right-hand side of Eq. (4), after using the Hamiltonian constraint equation, reads

$$\begin{aligned} A^{ij} \equiv & -\alpha \gamma^{1/2} (R^{ij} - \gamma^{ij} R) - \alpha \gamma^{-1/2} (2 \pi^{im} \pi_m^j - \pi \pi^{ij}) \\ & + \gamma^{1/2} (\alpha^{ij} - \gamma^{ij} \alpha^m_{|m}) + (\pi^{ij} \beta^m_{|m}) - \pi^{mj} \beta^i_{|m} \\ & - \pi^{mi} \beta^j_{|m} + \frac{8 \pi G}{c^4} \alpha \gamma^{1/2} (\gamma^{il} \gamma^{jm} T_{lm} - \gamma^{ij} \alpha^2 T^{00}). \end{aligned} \quad (15)$$

The longitudinal part of A^{ij} , namely, A_L^{ij} , may be written in terms of a vector V^i as

$$A_L^{ij} = V^i_{,j} + V^j_{,i} - \frac{2}{3} \delta_{ij} V^m_{,m} \quad (16)$$

which satisfies

$$\Delta V^i + \frac{1}{3} V^j_{,j} = A^{ij}_{,i}. \quad (17)$$

The elliptic equation that determines h_{ij}^{TT} , derivable using the condition $A_{TT}^{ij} = 0$, reads

$$\begin{aligned} 2 \gamma R_{ij} = & -\frac{2 \gamma^{1/2}}{\alpha} \gamma_{il} \gamma_{jn} [A_L^{ln} - (\pi^{ln} \beta^m_{|m}) + \pi^{lm} \beta^n_{|m} + \pi^{nm} \beta^l_{|m}] \\ & + \gamma_{ij} (2 \pi_n^m \pi_m^n - \pi^2) - 4 \pi_i^m \pi_{jm} + 2 \pi \pi_{ij} \\ & + \frac{2 \gamma}{\alpha} (\alpha_{|ij} - \gamma_{ij} \alpha^m_{|m}) + \frac{16 \pi G}{c^4} \gamma (T_{ij} + \gamma_{ij} \alpha^2 T^{00}). \end{aligned} \quad (18)$$

The left-hand side of the above equation, which introduces the Laplacian for h_{ij}^{TT} , takes the form

$$\begin{aligned} 2 \gamma R_{ij} = & -\chi^8 [\Delta h_{ij}^{TT} + \Delta (\psi^4) \delta_{ij} + (\psi^4)_{,ij}] \\ & + (\psi^4 h_{kl}^{TT} - h_{mk}^{TT} h_{ml}^{TT}) (\gamma_{kl,i} + \gamma_{ij,kl} - \gamma_{kj,il} - \gamma_{il,kj}) \\ & + 2 \gamma \gamma^{kl} \gamma^{np} (\Gamma_{n,il} \Gamma_{p,kj} - \Gamma_{n,ij} \Gamma_{p,kl}), \end{aligned} \quad (19)$$

where $\chi^8 = \psi^8 - \frac{1}{2} h_{mn}^{TT} h_{mn}^{TT}$ and where the Christoffel symbols $\Gamma_{i,jk}$ are given by $\frac{1}{2} (\gamma_{ij,k} + \gamma_{ik,j} - \gamma_{jk,i})$. For the derivation of Eq. (19), we have also used the relation $\gamma \gamma^{ij} = \chi^8 \delta_{ij} - \psi^4 h_{ij}^{TT} + h_{ik}^{TT} h_{kj}^{TT}$ [6]. In terms of a function ϕ , vanishing at spacelike infinity, ψ may be given by $(1 + \phi/8)$.

In the Hamiltonian formulation, the present approximation to Einstein's equations is identical with the replacement of only the evolution equation

$$\pi_{TT,0}^{ij} = -\frac{16 \pi G}{c^4} \frac{\delta H}{\delta h_{ij}^{TT}}, \quad (20)$$

which is the TT part of Eq. (4), through

$$\frac{\delta H}{\delta h_{ij}^{TT}} = 0. \quad (21)$$

The dropping of $\pi_{TT,0}^{ij}$ exactly corresponds to the dropping of the displacement current $(1/c)(\partial \mathbf{E}/\partial t)$ in the Maxwell-Ampère equation of electrodynamics. This is so since $-\mathbf{E}$ is the momentum conjugate in the Hamiltonian formulation of electrodynamics. However, this is not quite the way the MHD-like approximation to general relativity is implemented in [8], as they impose $h_{ij,0}^{TT} = 0$ in their scheme. In the post-Newtonian approximation, their scheme will not generate the correct leading order expression for π_{TT}^{ij} . Moreover, the dynamics associated with their approximation will coincide with the Einsteinian one through second post-Newtonian order only.

It is obvious that stationary spacetimes are reproduced exactly in our approximation. The constraint and the evolution equations, given by Eqs. (2), (3), (4), and (5), still determine ψ , π^i , α , and β^i , respectively, through elliptic equations. Moreover, Eq. (18) defines h_{ij}^{TT} , which describes the independent field degrees of freedom, through an elliptic equation [see Eq. (3.8a) in [6] for more details]. In addition, the longitudinal part of π_{ij}^{ij} , defined via V^i , can be determined through an elliptic equation for V^i , given by Eq. (17). The remaining quantities π_{TT}^{ij} should be determined, solely algebraically, using the evolution equation for γ_{ij} , given by Eq. (5), in the following manner. Let us write Eq. (5) in the form

$$\gamma_{ij,0} = B_{ij}, \quad (22)$$

where

$$B_{ij} \equiv \alpha \gamma^{-1/2} (2 \pi_{ij} - \gamma_{ij} \pi) + \beta_{i|j} + \beta_{j|i}. \quad (23)$$

Using the definition of γ_{ij} given by Eq. (7), we obtain

$$h_{ij,0}^{TT} = B_{ij} - \frac{1}{3} B_{ll} \delta_{ij}, \quad (24)$$

where $B_{ll} = 3 (\gamma^{mn} \gamma^{mn})^{-1} (B^{ll} - \gamma^{il} \gamma^{ik} h_{lk,0}^{TT})$ with B^{ll} given by $-\alpha \gamma^{-1/2} \gamma^{ll} \pi + 2 \beta^{l|l}$. Using these equations, π_{TT}^{ij} follows in terms of $h_{ij,0}^{TT}$, the only time derivative appearing in the minimal no-radiation approximation, given by

$$\begin{aligned} \pi_{TT}^{ij} = & -\tilde{\pi}^{ij} - \frac{\gamma^{1/2}}{\alpha} C^{ijkl} [\beta^{l|k} - \beta^m_{|m} \gamma^{ln} \gamma^{kn} (\gamma^{pq} \gamma^{pq})^{-1}] \\ & + \frac{\gamma^{1/2}}{2 \alpha} C^{ijkl} [\gamma^{lm} \gamma^{kn} - \gamma^{lr} \gamma^{kr} \gamma^{sm} \gamma^{sn} (\gamma^{pq} \gamma^{pq})^{-1}] h_{mn,0}^{TT}, \end{aligned} \quad (25)$$

where C^{ijkl} is the inverse matrix of the matrix

$$\begin{aligned} C_{ijkl}^{-1} \equiv & \frac{1}{2} [\delta_{il} \delta_{jk} + \delta_{ik} \delta_{jl} \\ & - (\gamma^{ij} - \gamma^{in} \gamma^{jn} \gamma^{mm} (\gamma^{pq} \gamma^{pq})^{-1}) h_{lk}^{TT}]. \end{aligned} \quad (26)$$

Notice that the matrix C_{ijkl}^{-1} deviates from the unit matrix in the quadratic order of h_{lk}^{TT} only. For the maximal slicing con-

dition $0 = \gamma^{ij} K_{ij} = \pi \gamma^{-1/2}/2$, the resulting C_{ijkl}^{-1} would be a unit matrix when applied to symmetric tensors.

The internal consistency of the proposed approach comes from the fact that only the evolution equations for the true field degrees of freedom are affected. This implies, in the evolution equations for the TT variables,

$$\pi_{TT,0}^{ij} = -\frac{16\pi G}{c^4} \frac{\delta H}{\delta h_{ij}^{TT}}, \quad (27a)$$

$$h_{ij,0}^{TT} = \frac{c^4}{16\pi G} \frac{\delta H}{\delta \pi_{TT}^{ij}}, \quad (27b)$$

if the first equation is solved for h_{ij}^{TT} using Eq. (18), then the second equation can be solved for π_{TT}^{ij} in terms of h_{ij}^{TT} , as given by Eq. (25). However, putting $\dot{h}_{ij}^{TT}=0$ in Eq. (27b) means destroying the Legendre transformation, which is a fundamental property of the physical theory, whereas setting $\dot{\pi}_{TT}^{ij}=0$ in Eq. (27a) implies a dynamical condition for the extremum (minimum) of energy. Imposing $h_{ij}^{TT}=0$, instead of solving for it, results in the well-known Wilson-Mathews approach, if π_{TT}^{ij} is determined through Eq. (22). We also note that a recent effort, the so-called CFC+ approach, which tries to improve conformal-flat approximations uses Eqs. (18) and (25), restricted to the leading post-Newtonian order, to get h_{ij}^{TT} and π_{TT}^{ij} [12]. This approach is identical with the full Einstein theory through the second post-Newtonian order.

In general, the Hamiltonian for the matter system is the Routh functional depending on the matter variables only [7,11],

$$\mathcal{R}[V_M, h_{ij}^{TT}(V_M), h_{ij,0}^{TT}(V_M)] = H - \frac{c^3}{16\pi G} \int d^3x \pi_{TT}^{ij} h_{ij,0}^{TT}. \quad (28)$$

In the stationary case, \mathcal{R} and H coincide. For our quasistationary approximation, we also shall take \mathcal{R} as the Hamiltonian. It agrees with the nondissipative part of the Einsteinian one through third post-Newtonian order as, e.g., given in Ref. [7].

Finally, we point out that \dot{h}_{ij}^{TT} is the *only* partial time derivative appearing in the coupled elliptic equations, present in the minimal no-radiation approximation. While employing any iterative procedure to solve our coupled elliptic equations, it should be natural to equate the (in general) small quantity \dot{h}_{ij}^{TT} to zero to obtain the first order solution to h_{ij}^{TT} and the rest of the variables involved. During the second stage of iteration, one should then employ \dot{h}_{ij}^{TT} , computed using the first order h_{ij}^{TT} , to get the second order solution to h_{ij}^{TT} and other quantities. In this manner, the iterative procedure may be extended to higher stages. The above procedure was employed, in the post-Newtonian framework, first to compute h_{ij}^{TT} to the second post-Newtonian order, which was employed to compute \dot{h}_{ij}^{TT} , required at the third post-Newtonian order calculations [7].

This communication will be followed by a detailed article [13], where the explicit expressions, in terms of the basic variables and their spatial derivatives, for the elliptic equations for ψ, α, β^i , and h_{ij}^{TT} as well as for π^i and V^i will be presented.

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